Elasticity, Thermal Expansion, Calorimetry , Heat Transfer

1. Two identical rods made of two different metals A and B with thermal conductivities K_A and K_B respectively are joined end to end. The free end of A is kept at a temperature T_1 while the free end of B is kept at a temperature T_2 ($< T_1$). Therefore, in the steady state **[NSEP-2017]**

(A) the temperature of the junction will be determined only by K_A and K_B

(B) if the lengths of the rods are doubled the rate of heat flow will be halved.

(C) if the temperature at the two free ends are interchanged the junction temperature will change

(D) the composite rod has an equivalent thermal conductivity of $\frac{2K_AK_B}{K_A+K_B}$

Ans. (BCD)

Sol. $$\frac{T_1 - T_j}{\frac{L}{k_A A}} = \frac{T_2 - T_j}{\frac{L}{k_B A}}$$

$\Rightarrow (T_1 - T_j)k_A = (T_2 - T_j)k_B$

So T_j depends on k_A, k_B & T_1, T_2

$$\frac{L}{k_A A} + \frac{L}{k_B A} = \frac{2L}{k_{eq} A} \Rightarrow k_{eq} = \frac{2k_A k_B}{k_A + k_B}$$

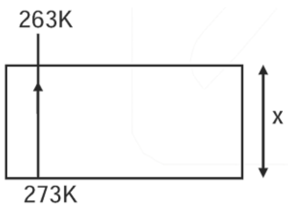
B, C, D are correct.

2. A small pond of depth 0.5 m deep is exposed to a cold winter with outside temperature of 263 K. Thermal conductivity of ice is $K = 2.2\ W\ m^{-1}\ K^{-1}$, latent heat $L = 3.4 \times 10^5\ Jkg^{-1}$ and density $\rho = 0.9 \times 10^3\ kgm^{-3}$. Take the temperature of the pond to be 273 K. The time taken for the whole pond to freeze is about. **[NSEP-2017]**

(A) 20 days (B) 25 days (C) 30 days (D) 35 days

Ans. (A)

Sol.



$$\frac{KA(10)}{x} = L\frac{d}{dt}(\rho A x)$$

$$\frac{2.2 \times (10)}{x} = 3.4 \times 10^5 \times 0.9 \times 10^3 \frac{dx}{dt}$$

$$\frac{22}{3.4 \times 0.9 \times 10^8}\int_0^t dt = \int_0^{0.5} x\,dx$$

$$\frac{22}{306\times10^6}t=\frac{1}{2}(0.5)^2$$

$$t=\frac{(0.5)^2\times306\times10^6}{44}\sec=\frac{306}{44\times4}\times10^6\sec$$

$$=\frac{306000\times10^3}{44\times4\times24\times3600}\approx20\text{ days}$$

3. Under standard conditions of temperature and pressure a piece of ice melts completely on heating it. Obviously, the increase in internal energy of the system (ice and water) is:

[NSEP-2018]

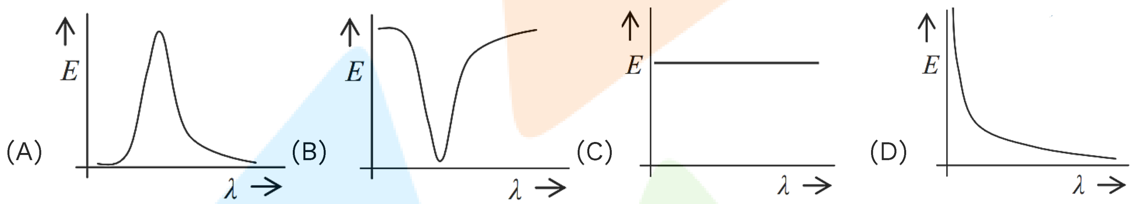
(A) equal to the heat given. (B) more than the heat given.
(C) less than the heat given. (D) zero.

Ans. (B)

Sol. On heating ice will convert into water and hence volume will decrease.
So change in internal energy will be more than heat supplied

4. Which of the following curves represents spectral distribution of energy of black body radiation?

[NSEP-2018]



Ans. (A)

Sol. Self explanatory by theory

5. A sphere and a cube having equal surface area are made of the same material. The two are heated to the same temperature and kept in identical surroundings. The ratio of their initial rates of cooling is: **[NSEP-2018]**

(A) 1 : 1 (B) $\sqrt{\frac{\pi}{2}}:1$ (C) $\sqrt{\frac{\pi}{3}}:1$ (D) $\sqrt{\frac{\pi}{6}}:1$

Ans. (D)

Sol. $$\frac{dQ}{dt}=e\sigma AT^4$$

$$ms\frac{dQ}{dt}=e\sigma AT^4$$

$$\frac{dQ}{dt}=\frac{e\sigma A}{ms}\left(T^4\right)$$

$$4\pi R^2=6a^2\Rightarrow\sqrt{\frac{4\pi}{6}}=\frac{a}{R}$$

$$Ms = \rho \frac{4}{3}\pi R^3$$

$$Mc = \rho a^3$$

$$\text{Ratio of Rate of cooling} = \frac{K_1}{K_2}$$

$$= \frac{A_1 / m_1}{A_2 / m_2}$$

$$= \frac{m_2}{m_1} = \frac{\rho a^3}{\rho \frac{4}{3}\pi R^3}$$

$$= \frac{a^3}{\frac{4}{3}\pi R^3} = \frac{1}{4\frac{\pi}{3}}\left(\frac{4\pi}{6}\right)^{3/2}$$

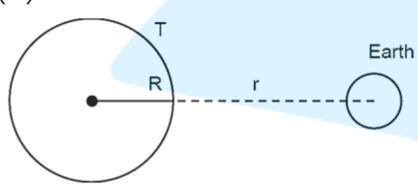
$$= \frac{\sqrt{\frac{4\pi}{6}\frac{4\pi}{6}\frac{4\pi}{6}}}{\sqrt{\frac{4\pi}{3}\frac{4\pi}{3}}} = \sqrt{\frac{\pi}{6}} : 1$$

6. The Sun having radius R and surface temperature T, emits radiation as a perfect emitter. The distance of the earth from the sun is r and the radius of the earth is R_e. The total radiant power incident on the earth is: **[NSEP-2018]**

(A) $\frac{R_e^2 R^2 \sigma T^4}{4\pi r^2}$ (B) $\frac{R_e^2 R^2 \sigma T^4}{r^2}$ (C) $\frac{4\pi R_e^2 R^2 \sigma T^4}{r^2}$ (D) $\frac{\pi R_e^2 R^2 \sigma T^4}{r^2}$

Ans. (D)

Sol.



Intensity at distance r form sun

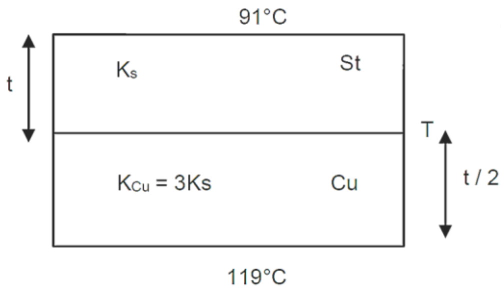
$$= \frac{\sigma T^4 4\pi R^2}{4n r^2}\pi R_e^2 \quad = \frac{\sigma T^4 \pi R^2 R_e^2}{r^2}$$

7. A steel cooking pan has copper coating at its bottom. The thickness of copper coating is half the thickness of steel bottom. The conductivity of copper is three times that of steel. If the temperature of blue flame is 119°C and that of the interior of the cooking pan is 91°C, then the temperature at the interface between the steel bottom and the copper coating in the steady state is: **[NSEP-2018]**

(A) 98°C (B) 103°C (C) 115°C (D) 108°C

Ans. (C)

Sol.



$$3K_sA\frac{119-T}{(t/2)} = \frac{AK_s\cdot(T-91)}{t}$$

$\Rightarrow 2 \times (357 - 3T) = (T - 91)$

$\Rightarrow 714 - 6T = T - 91$

$\Rightarrow 714 + 91 = 7T$

$$\Rightarrow T = \frac{805}{7} = 115^\circ C$$

8. Consider the following physical expressions **[NSEP-2019]**

(I) ρv^2 (ρ: density, v: velocity)

(II) $\frac{Y\Delta L}{L}$ (Y: Young's modulus, L: length)

(III) $\frac{\sigma^2}{\varepsilon_0}$ (σ: surface density of charge)

(iv) $h\rho rg$ (h: rise of a liquid in a capillary tube of radius r)

The expressions having same dimensional formula are

(A) I and II only
(B) II and III only
(C) II, III and IV only
(D) I, II and III only

Ans. (D)

Sol. $\rho v^2 \equiv \frac{F}{A}$

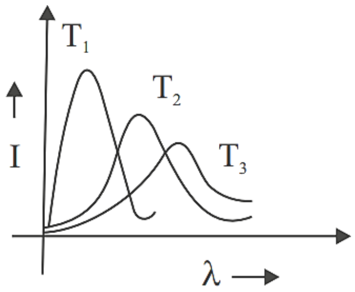
$y\frac{\Delta\ell}{\ell} \equiv \frac{F}{A}$

$h\rho rg \equiv T = \frac{F}{\ell}$

$\frac{\sigma^2}{\varepsilon_0} = \frac{F}{A}$

9. Plots of intensity (I) of radiation emitted by a black body versus wavelength (λ) at three different temperatures T_1, T_2 and T_3 respectively are shown in figure. Choose the correct statement.

[NSEP-2019]



(A) $T_1 > T_2 > T_3$ necessarily (B) $T_3 > T_2 > T_1$ necessarily

(C) $T_2 = (T_1 + T_3)/2$ necessarily (D) $T_2^2 > T_2 > T_3$ necessarily

Ans. (A)

Sol. $\lambda m\ T = C$

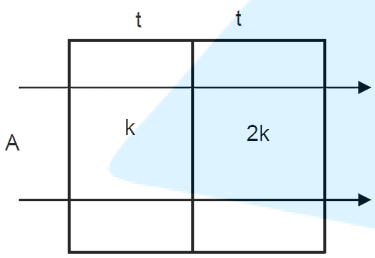
So $T_1 > T_2 > T_3$

10. Consider a composite slab consisting of two different materials having equal thickness and equal area of cross-section. The thermal conductivities are K and 2K respectively. The equivalent thermal conductivity of the composite slab is: **[NSEP-2019]**

(A) $\frac{2K}{3}$ (B) $\sqrt{2}$ K (C) 3K (D) $\frac{4K}{3}$

Ans. (D)

Sol.



$R_{eq} = R_1 + R_2$

$$\frac{2t}{k_{eq}A} = \frac{t}{kA} + \frac{t}{2kA}$$

$$\frac{2}{k_{eq}} = \frac{1}{k} + \frac{1}{2k}$$

$$k_{eq} = \frac{4k}{3}$$

11. The temperature of an isolated black body falls from T_1 to T_2 in time t. Then, t = C x where x is **[NSEP-2019]**

(A) $\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$ (B) $\left(\frac{1}{T_2^2} - \frac{1}{T_1^2}\right)$ (C) $\left(\frac{1}{T_2^3} - \frac{1}{T_1^3}\right)$ (D) $\left(\frac{1}{T_2^4} - \frac{1}{T_1^4}\right)$

Ans. (C)

Sol. $-Ms\frac{dT}{dt} = \sigma AT^4$

Let $\frac{\sigma A}{ms} = C$

$\frac{dT}{dt} = -C^1T^4$

$\int_{T_1}^{T_2}\frac{dT}{T^4} = -\int_0^t C^1dt$

$-\frac{1}{3}\left[\frac{1}{T_2^3} - \frac{1}{T_1^3}\right] = -C^1t$

$t = C\left[\frac{1}{T_2^3} - \frac{1}{T_1^3}\right]$

12. Heat is absorbed or evolved when current flows in a conductor having a temperature gradient. This phenomenon is known as **[NSEP-2019]**

(A) Joule effect (B) Peltier effect

(C) Seeback effect (D) Thomson effect

Ans. (D)

13. Three containers A, B and C are filled with water at different temperature. When 1 litre of water from A is mixed with 2 litre of water from B, the resulting temperature of mixture is $52^{\circ}C$. When 1 litre of water from B is mixed with 2 litre of water from C, the resulting temperature of mixture is $40^{\circ}C$. Similarly when 1 litre of water from C is mixed with 2 litre of water from A, the resulting temperature of mixture is $34^{\circ}C$. Temperature of mixture when one litre of water from each container is mixed (neglect the water equivalent of container) is **[NSEP-2020]**

(A) $40^{\circ}C$ (B) $42^{\circ}C$ (C) $38^{\circ}C$ (D) $45^{\circ}C$

Ans. (B)

Sol. Let θ_1, θ_2, and θ_3 and be the temperature of water in the three containers

When one litre from A and two litre from B is mixed, we get

$(\theta_1 - 52) = 2(52 - \theta_2) \Rightarrow \theta_1 + 2\theta_2 = 3\times 52$

When one litre from B and two litre from C is mixed, we get

$(\theta_2 - 40) = 2(40 - \theta_3) \Rightarrow \theta_2 + 2\theta_3 = 3\times 40$

When one litre from C and two litre from A is mixed, we get

$(\theta_3 - 34) = 2(34 - \theta_1) \Rightarrow \theta_3 + 2\theta_1 = 3\times 34$

Adding the three, we get $3(\theta_1 + \theta_2 + \theta_3) = 3\times(52 + 40 + 34) \Rightarrow \theta_1 + \theta_2 + \theta_3 = 126$ If θ_0 is the temperature when one litre from each A, B and C is mixed then

$(\theta_1 - \theta_0) = (\theta_0 - \theta_2) + (\theta_0 - \theta_3) \Rightarrow \theta_1 + \theta_2 + \theta_3 = 3\theta_0 \Rightarrow \theta_0 = 42^{\circ}$

14. What amount of heat will be generated in a coil of resistance R(ohm) due to a total charge Q (coulomb) passing through it if the current in the coil decreases down to zero halving its value every Δt second? **[NSEP-2020]**

(A) $\frac{1}{2}\frac{Q^2R}{\Delta t}$ (B) $\frac{Q^2R}{\Delta t}\ln 2$ (C) $\frac{1}{2}\frac{Q^2R}{\Delta t}\ln 2$ (D) $\frac{1}{4}\frac{Q^2R}{\Delta t}$

Ans. (C)

Sol. Let the initial current through the coil be I_0 at $t = 0$. The current decrease down to zero halving after each Δt second This means that the current at any time t is expressed as $i = I_0 e^{-\frac{\ln 2}{\Delta t}t}$. The heat produced in time dt in the coil is $dH = i^2Rdt$. The total heat produced be $H = \int_0^\infty i^2Rdt$

Substituting the values

$$H = \int_0^\infty I_0^2 e^{-2\frac{\ln 2}{\Delta t}t} Rdt = I_0^2R\int_0^\infty e^{-2\frac{\ln 2}{\Delta t}t}dt = \frac{I_0^2R\Delta t}{-2\ln 2}\left\{e^{-2\frac{\ln 2}{\Delta t}t}\Big|_0^\infty\right\} = \frac{I_0^2R\Delta t}{2\ln 2} \text{.........(1) Also we know}$$

$$i = \frac{dQ}{dt} \quad \text{or} \quad dQ = idt \text{ or } Q = \int_0^q dQ = \int_0^\infty idt = \int_0^\infty I_0 e^{-\frac{\ln 2}{\Delta t}t}dt = \left\{-\frac{I_0\Delta t}{\ln 2}e^{-\frac{\ln 2}{\Delta t}t}\Big|_0^\infty\right\} = \frac{I_0\Delta t}{\ln 2} \text{ or}$$

$$I_0 = \frac{Q\ln 2}{\Delta t} \text{ Substituting in (1) } H = \frac{\left(\frac{Q\ln 2}{\Delta t}\right)^2 R\Delta t}{2\ln 2} = \frac{1}{2}\frac{Q^2R}{\Delta t}\ln 2$$

15. Consider the process of the melting of a spherical ball of ice originally at 0° Assuming that the heat is being absorbed uniformly through the surface and the rate of absorption is proportional to the instantaneous surface area. Which of the following is true for the radius (r) of the ice ball at any instant of time? Assume that the initial radius of the ice ball at $t = 0$ is $r = R_0$ and that the shape of the ball always remains spherical during melting. Also assume that L and ρ are respectively the latent heat and density of ice at 0° **[NSEP-2021]**

(A) radius decreases exponentially with time as $r = R_0 e^{-\frac{kt}{\rho L}}$. Here k is constant

(B) radius decreases exponentially with time as $r = R_0 e^{-\frac{k\rho t}{2L}}$

(C) radius of the ice ball decreases with time linearly with a slope $-\frac{k}{\rho L}$

(D) radius of the ice ball decreases with time linearly with a slope $-\frac{k\rho}{2L}$

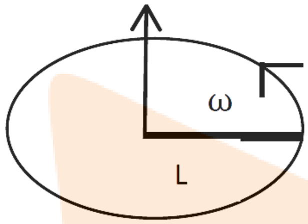
Ans. (C)

Sol. The instantaneous rate of absorption of heat is $\frac{dQ}{dt} \propto 4\pi r^2 = k4\pi r^2$ Also $\frac{dQ}{dt} = -L\frac{dm}{dt}$ So

$-L\frac{dm}{dt}=k4\pi r^2$ or $-L\frac{d}{dt}\left(\frac{4\pi}{3}r^3\rho\right)=k4\pi r^2$ or $\frac{dr}{dt}=-\frac{k}{\rho L}\Rightarrow\int_{r_0}^{r}dr=-\frac{k}{\rho L}\int_0^t dt\Rightarrow r=r_0-\frac{k}{\rho L}t$ or

$r=-\frac{k}{\rho L}t+r_0$ which is a straight line with negative slope $=-\frac{k}{\rho L}$ where k is constant

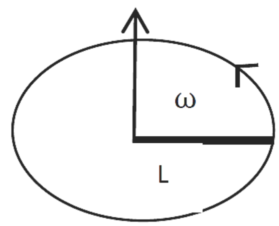
16. A thin uniform metallic rod of length L and radius R rotates with an angular velocity ω in a horizontal plane about a vertical axis passing through one of its ends. The density and the Young's modulus of the material of the rod are ρ and Y respectively. The elongation in its length is **[NSEP-2021]**



(A) $\frac{\rho\omega^2L^3}{6Y}$ (B) $\frac{\rho\omega^2L^3}{3Y}$ (C) $\frac{\rho\omega^2RL^2}{2Y}$ (D) $\frac{\rho\omega^2L^3}{2Y}$

Ans. (B)

Sol. When the rod rotates about a vertical axis through one of its ends, every point on the rod experiences a centrifugal force. If we consider a small length dx of mass λdx at a distance x from the axis where $\lambda=\frac{M}{L}=\frac{\pi R^2L\rho}{L}=\pi R^2\rho$,



The outward pull on this length x is $=\frac{\lambda dx\omega^2x^2}{x}=T(\text{say})$

This force will cause an elongation in the rod, because of its elasticity.

The elongation may be given by $d\xi=\frac{Tx}{AY}=\frac{\lambda dx\omega^2x}{\pi R^2Y}\cdot x=\frac{\rho\omega^2}{Y}\cdot x^2dx$.

The total elongation in the rod is therefore $\int d\xi=\frac{\rho\omega^2}{Y}\int_0^L x^2dx=\frac{\rho\omega^2L^3}{3Y}$.

17. A certain rod of uniform area of cross section $A\left(A=1.0\text{cm}^2\right)$ with its length $=2\text{m}$ is thermally insulated on its lateral surface. The thermal conductivity (K) of the material of the rod varies with temperature T as $K=\frac{\alpha}{T}$ where α is a constant. The two ends of the rod are maintained at temperature of $T_1=90^\circ$ and $T_2=10^\circ$... The correct option(s) is /are **[NSEP-2021]**

(A) The temperature at 50cm from the colder end is 17.32° -

(B) The temperature at 50cm from the hotter end is 51.96° -

(C) The rate of heat flow per unit area of cross section of the rod is 1.1α in SI units.

(D) The temperature gradient is numerically higher near the hot end compare to that near the cold end.

Ans. (ABCD)

Sol The rate of flow of heat in a solid rod is expressed as the thermal current

$H = \frac{dQ}{dt} = KA\left(-\frac{dT}{dx}\right)$ Given that the thermal conductivity $K = \frac{\alpha}{T}$ so one can write

$H\int_0^l dx = -\alpha A\int_{90}^{10}\frac{dT}{T} \Rightarrow Hl = -\alpha A \ln\frac{10}{90} = 2\alpha A\ln 3$ using l = 2m we get

$H = \alpha A\ln 3 = 1.1\alpha$ A Further at any intermediate location at a distance x from hot end

$H\int_0^x dx = -\alpha A\int_{90}^{T}\frac{dT}{T} \Rightarrow Hx = -\alpha A\ln\frac{T}{90} \Rightarrow T = 90\times e^{-Hx/\alpha A}$

At $x = 0.5m, T = 90\times e^{-Hx/\alpha A} = 90e^{-1.1\times 0.5} = 51.96^0 C$ also

At $x = 1.5m, T = 90e^{-1.1\times 1.5} = 17.32^\circ C$ The temperature gradient may be expressed as

$\frac{dT}{dx} = -\frac{TH}{\alpha A}$ is higher near the hotter end than that near the colder end.

18. Solar constant for Earth is 2.0 cal per cm^2 per minute. [1cal = 4.2 J]. Angular diameter of the Sun (as seen from the Earth) is $\frac{1^\circ}{2}$ (= half a degree). Treating Sun as a black body, its surface temperature is estimated to be nearly **[NSEP-2022]**

(A) 6000 K (B) 5800 K (C) 6200 K (D) 5500 K

Ans. (A)

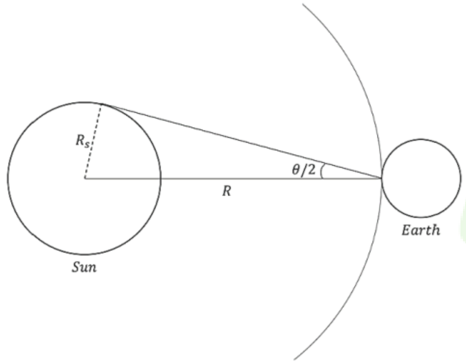
Sol. Solar constant = $\frac{\text{Energy}}{\text{Area}\times\text{time}} = \frac{\text{power}}{\text{surface area}}$

$\therefore$ Solar constant, S = $\frac{\sigma\times 4\pi R_S^2\times T^4}{4\pi R^2}$

R_S = Radius of Sun

R = Mean orbital radius of Earth

S = $\frac{\sigma\times R_S^2\times T^4}{R^2}$ ...(1)



$180^\circ \to \pi$

$1^\circ \to \frac{\pi}{180}$

$\left(\frac{1}{2}\right)^\circ \to \frac{\pi}{180}\times\frac{1}{2}$

$\theta = \dfrac{\pi}{360}$ radian

As θ is small, we can assume

$\tan\theta \sim \theta$

$\Rightarrow \tan\dfrac{\theta}{2} = \dfrac{R_s}{R} \Rightarrow \dfrac{\theta}{2} = \dfrac{R_s}{R}$ ...(2)

$\therefore$ from eqn. (1) & (2),

$$S = \sigma \times \frac{\theta^2}{4} \times T^4$$

$$\therefore T^4 = \left(\frac{4 \times S}{\sigma\theta^2}\right)$$

$$T^4 = \left(\frac{4 \times 2 \times 4.2 \times 10^4 \times 360^2}{60 \times 5.67 \times 10^{-8} \times \pi^2}\right)$$

$T \cong 6000K$

19. End A of a uniform thin rod of length 2L is in boiling water (100ºC and end B is in melting ice (0ºC). P and Q are two points at distance $\dfrac{L}{2}$ from A and B respectively. A similar bent rod of length $\dfrac{3L}{2}$ of same material and equal cross section is joined to rod AB between points P and Q as shown in figure. Then **[NSEP-2022]**

(A) Temperature at P will increase and that at Q will decrease

(B) Rate of flow of heat will increase by 25%

(C) Rate of flow of heat will decrease by 20%

(D) Rate of heat flow will increase by 37.5T

Ans. (B)

Sol. Let the thermal conductivity of material is K and cross-section area is A.

Before joining the bent rod

$T_P = 75ºC, T_Q = 25ºC$ and $\dfrac{dQ}{dt} = \dfrac{T_0 KA}{2L}$

After joining the bent rod net thermal resistance of combination is

$$R_{Th} = \frac{L}{2KA} + \frac{\frac{3L}{2KA} \times \frac{L}{KA}}{\frac{3L}{2KA} + \frac{L}{KA}} + \frac{L}{2KA} = \frac{L}{KA} + \frac{3L}{5KA} = \frac{8L}{5KA}$$

$$\frac{dQ}{dt} = \frac{5T_0 KA}{8L}$$

So, percentage change in rate of heat flow

$$= \frac{\frac{5T_0KA}{8L} - \frac{T_0KA}{2L}}{\frac{T_0KA}{2L}} \times 100 = 25\%$$

20. A uniform rod of the material of Young's modulus Y is pushed over a smooth horizontal surface by a constant horizontal force F. The area of cross-section of the rod is A. The compressional strain in the rod is **[NSEP-2022]**

(A) $\frac{F}{AY}$ (B) $\frac{F}{2AY}$ (C) $\frac{3F}{2AY}$ (D) $\frac{2F}{AY}$

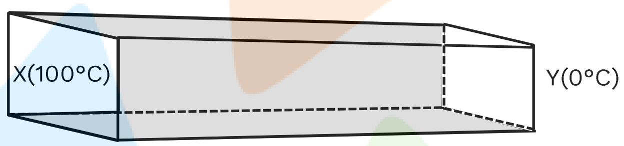
Ans. (B)

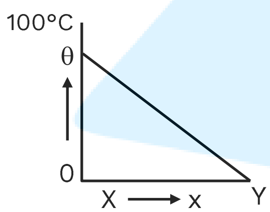
Sol. When force is applied at one end, we know that extension

$$\Delta l = \frac{Fl}{2AY}$$

$$\Rightarrow \text{Average strin} = \frac{\Delta l}{l} = \frac{F}{2AY}$$

21. The figure shows a lagged bar XY of non-uniform cross section. One end X of the bar is maintained at $100^\circ C$ and the other end Y at $0^\circ C$. The variation of temperature along its length from X to Y in steady state is best represented by the curve. **[NSEP-2023]**



(A)


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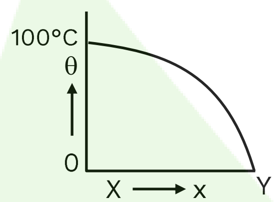

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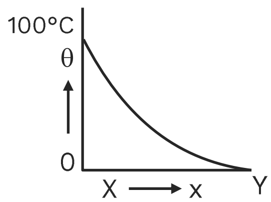

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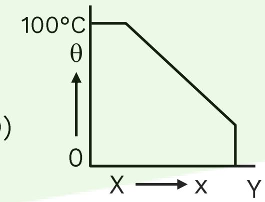


Ans. **(B)**

Sol.

$$\frac{dQ}{dt} = -kA\frac{dQ}{dx}$$

$$\frac{dQ}{dx} = -\frac{1}{kA} \times \frac{dQ}{dt}$$

$$\frac{dQ}{dx} \propto -\frac{1}{A}$$

22. Assuming the Sun to be a spherical body (radius R_S) of surface temperature T, the total radiation power received by Earth (radius R_E) at a distance r from Sun is **[NSEP-2023]**

(A) $\frac{\sigma\pi R_E^2 R_s^2 T^4}{r^2}$ (B) $\frac{\sigma 4\pi R_E^2 R_S^2 T^4}{r^2}$ (C) $\frac{\sigma\pi R_E^2 R_S^2 T^4}{4r^2}$ (D) $\frac{\sigma R_E^2 R_s^2 T^4}{4\pi r^2}$

Ans. **(A)**

Sol. $I = \frac{P_S}{4\pi r^2} = \frac{e\sigma 4\pi R_S^2 T_S^4}{4\pi r^2}$

$e = 1$

power received by earth

$P = I.\pi R_e^2$

$= \frac{\sigma 4\pi R_S^2 T_S^4}{4\pi r^2}.\pi R_e^2$

$= \frac{\sigma.R_S^2 T_S^4}{r^2}.\pi R_e^2$

KTG And Thermodynamics

1. If a system is made to undergo a change from an initial state to a final state by adiabatic process only, then **[NSEP-2017]**

(A) the work done is different for different paths connecting the two states

(B) there is no work done since there is no transfer of heat

(C) the internal energy of the system will change

(D) the work done is the same for all adiabatic paths.

Ans. (CD)

Sol. Adiabatic process $\Delta Q = 0$ du = –dw du : depends on initial and final position.

(C, D)

2. Two moles of hydrogen are mixed with n moles of helium. The root mean square speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then n is. **[NSEP-2017]**

(A) 3 (B) 2 (C) 1.5 (D) 2.5

Ans. (B)

Sol.

$$V_{rms} = \sqrt{\frac{3RT}{M_{mix}}}$$

$$V_{sound} = \sqrt{\frac{\gamma RT}{M_{mix.}}}$$

$$V_{rms} = \sqrt{2}\,V_{sound}$$

$$\sqrt{\frac{3RT}{M_{mix}}} = \sqrt{2}\sqrt{\frac{\gamma RT}{M_{mix}}}$$

$$r = \frac{3}{2}$$

$$r_{mix} = \frac{n_1C_{P_1} + n_2C_{P_2}}{n_1C_{v_1} + n_2C_{v_2}}$$

$$\frac{3}{2} = \frac{2\times\frac{7R}{2} + n\times\frac{5R}{2}}{2\times\frac{5R}{2} + n\times\frac{3R}{2}} \Rightarrow \frac{3}{2} = \frac{14+5n}{10+3n} \Rightarrow 30 + 9n = 28 + 10n \Rightarrow n = 2$$

3. The molar specific heat of an ideal gas in a certain thermodynamic process is $\frac{\alpha}{T}$ where α is a constant. If the adiabatic exponent is $\gamma = \frac{c_p}{c_v}$, the work done in heating the gas from T_0 to nT_0 is. **[NSEP-2018]**

(A) $\frac{1}{\alpha}\ell nn$

(B) $\alpha\ell nn - \frac{(n-1)}{(\gamma-1)}RT_0$

(C) $\alpha\ell nn - (\gamma-1)RT_0$

(D) $\frac{(n-1)}{(\gamma-1)}RT_0$

Ans. (B)

Sol. $dQ = CdT = \frac{\alpha}{T}dT$

$\Delta Q = \alpha \ell nn$

$W = \Delta Q - \Delta U = \alpha \ln n - \frac{R(n-1)T_0}{\gamma - 1}$

4. A horizontal insulated cylinder of volume V is divided into four identical compartments by stationary semi-permeable thin partitions as shown. The four compartments from left are initially filled with 28 g helium, 160 g oxygen, 28 g nitrogen and 20 g hydrogen respectively. The left partition lets through hydrogen, nitrogen and helium while the right partition lets through hydrogen only. The middle partition lets through hydrogen and nitrogen both. The temperature T inside the entire cylinder is maintained constant. After the system is set in equilibrium,

[NSEP-2018]

He	O_2	N_2	H_2

(A) pressure of helium is $\frac{14RT}{V}$

(B) pressure of oxygen is $\frac{20RT}{V}$

(C) pressure of nitrogen is $\frac{4RT}{3V}$

(D) pressure of hydrogen is $\frac{10RT}{V}$

Ans. (ABCD)

28 g He $n_1 = 7$	160 g O_2 $n_2 = 5$	28 g N_2 $n_3 = 1$	20 g H_2 $n_4 = 10$

Sol.

After

He H_2 N_2	He H_2 N_2	– H_2 N_2	– H_2

$P_{He} = \frac{n_1RT}{V/2} = \frac{7RT}{V/2} = \frac{14RT}{V}$

$P_{O_2} = \frac{n_2RT}{V/4} = \frac{5RT}{V/4} = \frac{20RT}{V}$

$P_{N_2} = \frac{n_3RT}{\frac{3}{4}V} = \frac{4}{3}\frac{RT}{V}$

$P_{H_2} = \frac{n_4RT}{V} = \frac{10RT}{V}$

5. The average translational kinetic energy of oxygen (M = 32) molecules at a certain temperature is 0.048 eV. The translational kinetic energy of nitrogen (M = 28) molecules at the same temperature is (consider the two gases to be ideal) **[NSEP-2019]**

(A) 0.0015 eV (B) 0.042 eV

(C) 0.048 eV (D) 0.768 eV

Ans. (C)

Sol. $\langle KE\rangle_T = 3/2\ kT$

Independent of molar mass

6. If the average mass of a smoke particle in an Indian kitchen 3×10^{-17} kg, the rms speed of the smoke particles at 27°C is approximately **[NSEP-2019]**

(A) 2 cm/sec (B) 2 m/sec

(C) 2 km/sec (D) none of these

Ans. (A)

Sol. $\sqrt{\frac{2\times 25}{3}\times\frac{300}{3\times 10^{-17}}}$ $\sqrt{\frac{3\times 25}{3}\times\frac{300}{3\times 10^{-17}}}$

$5 \times 10^{+9} \times 3.1$

$15 \times 10^{+9}$

$$\sqrt{\frac{3\times 25\times 300}{3\times 6.022\times 10^{23}\times 3}} = 2\text{cm}/\text{sec}$$

7. In a real gas **[NSEP-2019]**

(A) the force of attraction between the molecules depends upon intermolecular distance.

(B) internal energy depends only upon temperature.

(C) internal energy is a function of both temperature and volume.

(D) internal energy is a function of both temperature and pressure.

Ans. (ACD)

8. One mole of a gas with $\gamma = \frac{5}{3}$ is mixed with two moles of another non-interacting gas with $\gamma = \frac{7}{5}$

The ratio of specific heats $\gamma = \frac{C_P}{C_V}$ of mixture is approximately **[NSEP-2020]**

(A) 1.50 (B) 1.46 (C) 1.49 (D) 1.53

Ans. (B)

Sol. Knowing that $C_V = \frac{R}{(\gamma-1)}, C_P = \frac{\gamma R}{(\gamma-1)}$ and $\gamma_{mix} = \frac{n_1C_{P1}+n_2C_{P2}}{n_1C_{V1}+n_2C_{V2}}$ with

$\gamma_1 = \frac{5}{3}$ and $\gamma_2 = \frac{7}{5}$ and $n_1 = 1, \& n_2 = 2$, we get

$C_{V1} = \frac{3}{2}R, C_{P1} = \frac{5}{2}R$ and $C_{V2} = \frac{5}{2}R, C_{P2} = \frac{7}{2}R$ and obtain

$$\gamma_{mix} = \frac{n_1C_{P1} + n_2C_{P2}}{n_1C_{V1} + n_2C_{V2}} = \frac{19}{13} = 1.46$$

9. An ideal gas is expanding such that $PT^3 =$ constant. The coefficient of volume expansion of the gas is **[NSEP-2020]**

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{4}{T}$

Ans. (D)

Sol. Given that $PT^3 = K$, using $PV = \mu RT$ or $P = \frac{\mu RT}{V}$ we get

$\frac{\mu RT}{V}T^3 = K$ or $T^4 = \frac{KV}{\mu R}$ differentiating we get $4T^3dT = \frac{KdV}{\mu R}$ thereby

$\frac{dV}{dT} = \frac{4\mu RT \cdot T^2}{K} = \frac{4V}{T}$ so coefficient of volume expansion $\frac{1}{V}\frac{dV}{dT} = \frac{4}{T}$

10. An ideal monatomic gas is confined within a cylinder by a spring loaded piston of cross-sectional area $4 \times 10^{-3} m^2$. Initially the gas is at $400K$ and occupies a volume $2 \times 10^{-3} m^3$ and the spring is in its relaxed position. The gas is heated by an electric heater for some time. During this time the gas expands and the piston moves out by a distance $0.1m$. The spring connected to the rigid wall is massless and frictionless. The force constant of the spring is $2000Nm^{-1}$ and atmospheric pressure is $10^5 Nm^{-2}$ then **[NSEP-2020]**

(A) The final temperature of the gas is $720K$.

(B) The work done by gas in expanding is $50J$

(C) The heat supplied by heater is $190J$

(D) The heat supplied by heater is $290J$

Ans. (ABD)

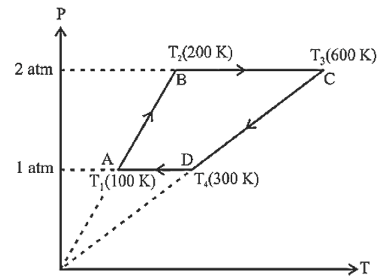
Sol. The given parameters are $P_i = 1 \times 10^5 N/m^2, V_i = 2 \times 10^{-3} m^3$,

$P_f = P_o + \frac{kx}{A} = 1.5 \times 10^5 N/m^2$ and $V_f = V_0 + Ax = 2.4 \times 10^{-3} m^3$ Now $T_f = \frac{P_F \times V_f}{P_iV_i}T_i = 720K$

And $\Delta U = nC_VdT = \frac{nR}{\gamma - 1}dT = \frac{P_iV_i}{\gamma - 1}\frac{dT}{T} = 240J$ $\Delta W = P_0Ax + \frac{1}{2}kx^2 = 50J$ then

$\Delta Q = \Delta U + \Delta W = 290J$

11. The work done by the three moles of an ideal gas in the cyclic process ABCD shown in the diagram is approximately. Given that **[NSEP-2021]**



T_1 = 100 K, T_2 = 200 K and T_3 = 600 K, T_4 = 300 K

(A) 7.5 kJ (B) 5.0 kJ (C) 2.5 kJ (D) Zero

Ans. (B)

Sol. The process from A to B is isochoric $\because P \propto T$ means the volume is constant. Therefore the work done $dW_{AB} = PdV = 0$ From B to C the process is isobaric so work done is $dW_{BC} = PdV = nRdT = 3 \times R \times (600 - 200) = 1200R$. CD is again isochoric process so work done $W_{CD} = 0$. Further the process from D to A is isobaric means P constant and work done is $dW_{DA} = PdV = nRdT$ or $dW_{DA} = 3 \times R(100 - 300) = -600R$. Thus the total work done is $W = 1200R - 600R = 600R = 4986J = 4.986kJ = 5.0kJ$

12. The molar specific heat capacity of a certain gas is expressed as $C = C_V + \alpha\frac{P}{T}$. The equation of state for the process can be written as (α and Aare constant) **[NSEP-2021]**

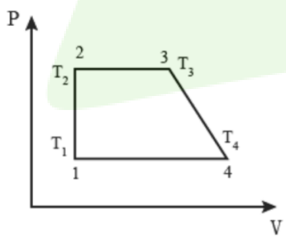
(A) $PV = RT$ (B) $V = \alpha T^2$ (C) $V^2 = \alpha \ln T$ (D) $T = Ae^{\frac{V}{\alpha}}$

Ans. (D)

Sol. From first law of thermodynamics $dQ = dU + dW \Rightarrow CdT = C_V dT + PdV$. Given that $C = C_V + \alpha\frac{P}{T}$

Substituting the value we get $\left(C_V + \alpha\frac{P}{T}\right)dT = C_V dT + PdV \Rightarrow \alpha\frac{dT}{T} = dV$ on integration we get $\alpha \times \ell nT = V +$ constant or $T = Ae^{V/\alpha}$ Ans: d

13. A cyclic process 1-2-3-4-1 consisting of two isobars 2 – 3 and 4 – 1, an isochor 1 – 2 and a process 3 – 4 represented by straight line on a P – V diagram, as shown in figure, involves n moles of an ideal gas. The gas temperatures at states 1,2,3 & 4 are T_1, T_2, T_3 and T_4 respectively. Also points 3 and 4 lie on the same isotherm. The work done by gas during the cycle is **[NSEP-2022]**

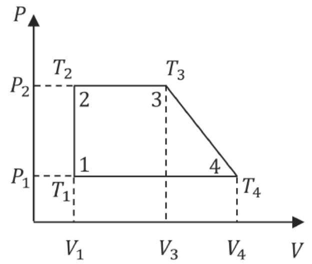


(A) $\frac{1}{2}nR(T_2 - T_1)\left(\frac{T_2}{T_1} + \frac{T_3}{T_4} - 2\right)$ (B) $\frac{1}{2}nR(T_3 - T_2)\left(\frac{T_3}{T_2} + \frac{T_4}{T_1} - 2\right)$

(C) $\frac{1}{2}nR(T_2 - T_1)\left(\frac{T_3}{T_1} + \frac{T_3}{T_2} - 2\right)$ (D) Zero

Sol. (C)

Ans.



Given $T_3 = T_4, V_1 = V_2, P_1 = P_4$ and $P_2 = P_3$

$\Rightarrow$ Work done which is area of trapezium $= \frac{1}{2}\times\left[(V_4 - V_1)+(V_3 - V_2)\right](P_2 - P_1)$

$$= \frac{1}{2}\times\left[V_4 + V_3 - 2V_2\right]\times(P_2 - P_1)$$

$$= \frac{1}{2}\times\left[P_2V_2 - P_1V_2\right]\left[\frac{V_4}{V_2}+\frac{V_3}{V_2}-2\right]$$

$$= \frac{1}{2}\times\left[P_2V_2 - P_1V_1\right]\left[\frac{V_4}{V_2}+\frac{V_3}{V_2}-2\right]$$

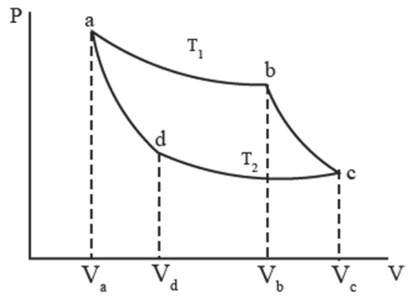
Using PV = nRT,

$$= \frac{1}{2}\times\left[nRT_2 - nRT_1\right]\left[\frac{V_4}{V_1}+\frac{V_3}{V_2}-2\right]$$

$$= \frac{1}{2}\times nR\left[T_2 - T_1\right]\left[\frac{T_4}{T_1}+\frac{T_3}{T_2}-2\right]$$

$$= \frac{1}{2}\times nR\left[T_2 - T_1\right]\left[\frac{T_3}{T_1}+\frac{T_3}{T_2}-2\right]$$

14. Two identical Carnot (cycles) engines oprate between maximum and minimum temperatures T_1 and T_2 volume limits, V_a, V_b, V_c, & V_d as shown in figure. Given that $\frac{V_c}{V_a} = e^3$ and $\frac{T_1}{T_2} = e$ (e is the base of natural logarithm). Engine 1 operates on mono atomic gas while engine 2 on diatomic gas. Choose correct alternatives **[NSEP-2022]**



(A) Ratio of volumes $\frac{V_{b,1}}{V_{b,2}} = e$

(B) Ratio of work done per cycle of the two is $\frac{W_1}{W_2} = 3$

(C) Ratio of work done per cycle for the two is $\frac{W_1}{W_2} = 1$

(D) Ratio of efficiencies (η) of two engines $\frac{\eta_1}{\eta_2} = 1$

Ans. (ABD)

Sol. Efficiency in both cases remains same as efficiency depends on temperature only.

$\therefore$ Option D is correct.

For adiabatic expansion,

$T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1}$

For Engine 1,

$$\Rightarrow eT_2\left(V_{b1}\right)^{\frac{2}{3}} = T_2\left(e^3V_a\right)^{\frac{2}{3}}$$

$$\Rightarrow V_{b1} = e^{\frac{3}{2}}V_a$$

Similarly, for Engine 2,

$$V_{b2} = e^{\frac{1}{2}}V_a$$

$$\Rightarrow \frac{V_{b1}}{V_{b2}} = e$$

$\therefore$ Option A is correct.

Work done by the engine per cycles is given by,

$W = \eta Q_{abs}$

Heat is absorbed by the engine during isothermal expansion,

$$\Rightarrow Q_{abs} = nRT_1 \ln\frac{V_b}{V_a}$$

$$\Rightarrow W = hnRT_1 \ln\frac{V_b}{V_a}$$

$$\frac{W_1}{W_2} = \frac{\ln\frac{V_{b1}}{V_a}}{\ln\frac{V_{b2}}{V_a}} = \frac{\ln e^{\frac{3}{2}}}{\ln e^{\frac{1}{2}}} = 3$$

$\therefore$ Option B is correct.

15. Two Carnot heat engines are connected in series such that the sink of the first engine is heat source of the second. Efficiency of the engines are η_1 and η_2 respectively. Net efficiency η of the combination is given by **[NSEP-2023]**

(A) $\eta = \eta_1 + \eta_2$ (B) $\eta = \frac{\eta_1\eta_2}{\eta_1 + \eta_2}$ (C) $\eta = \eta_1 + \eta_2(1-\eta_1)$ (D) $\eta = \eta_1 - \eta_2(1-\eta_1)$

Ans. **(C)**

Sol. $\frac{Q_2}{Q_1} = 1 - \eta_1$

$\frac{Q_3}{Q_2} = 1 - \eta_2$

$\eta_1 = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$\eta_2 = \frac{Q_2 - Q_3}{Q_2} = 1 - \frac{Q_3}{Q_2}$

$\eta_{net} = \frac{Q_1 - Q_2 + Q_2 - Q_3}{Q_1}$

$= 1 - \frac{Q_2}{Q_1} + \frac{Q_2}{Q_1} - \frac{Q_3}{Q_1} = \eta_1 + \eta_2(1-\eta_1)$

16. An ideal gas (n moles) is initially at pressure P and temperature T. It is cooled isochorically to a pressure $\frac{P}{4}$. The gas is then expanded at a constant pressure so as to attain back its initial temperature T. Work done by gas during the entire process is **[NSEP-2023]**

(A) $\frac{5}{4}$nRT (B) $\frac{3}{4}$nRT (C) $\frac{1}{4}$nRT (D) Zero **Ans.** **(B)**

Sol. PV = nRT

$P \rightarrow \frac{P}{4}$

$T \rightarrow \frac{T}{4}$

PV = nRT

$\frac{T}{4} \rightarrow T$

$V \rightarrow 4V$

$W = \frac{P}{4}.3V = \frac{3}{4}PV = \frac{3}{4}nRT$

17. Two moles of nitrogen in a container, of negligible thermal capacity, are initially at 17°C. The gas is compressed adiabatically from an initial volume of 120 liter to 80 liter. The correct option(s) is/are **[NSEP-2023]**

(A) Initial pressure of the gas is nearly 40.2kPa

(B) Final temperature of the gas is nearly 68°C

(C) Work done by the gas is 2.12 kJ

(D) The internal energy of the gas increases by 2.12 kJ

Ans. **(ABD)**

Sol. n = 2

T_i = 290 k

Adibatic process

V_i = 120 ltr = 0.12 m^3

V_f = 80 ltr = 0.08 m^3

(A) $P_iV_i = nRT_i$

$$P_i = \frac{2 \times 8.314 \times 290}{0.12} = 40184 \text{ Pa}$$

P_i = 40.2 kPa

(B) $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$$290(0.12)^{2/5} = T_f(0.08)^{2/5}$$

$$T_f = 290\left(\frac{0.12}{0.08}\right)^{2/5}$$

$$T_f = 341\text{K} \simeq 68°\text{C}$$

(C) $W = \frac{nR\Delta T}{1-\gamma} = \frac{-5}{2}nR\Delta T$

$$W = \frac{-5}{2} \times 2 \times 8.314 \times 51$$

= −2120 J

W = −2.12 KJ

(D) $\Delta U = \frac{f}{2}nR\Delta T = -w$

ΔU = 2.12 KJ

$\because$ ΔU is +ve

$\Rightarrow$ U increases